MAT 1332B, Calculus for Life Sciences II
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Corrections to the textbook chapter 4.7

Exercise 4.7.12

Since $\sqrt{t}+t > \sqrt{t}$ for $t > 0$, we have the inequality

$$\int_0^1 \frac{1}{\sqrt{t}+t}dt < \int_0^1 \frac{1}{\sqrt{t}}dt = \lim_{a\to 0}\int_a^1 \frac{1}{\sqrt{t}}dt = \lim_{a\to 0} 2\sqrt{t}|_a^1 = \lim_{a\to 0} 2\left(1-\sqrt{a}\right) = 2.$$

Therefore the first integral on the left converges.

4.7, Exercise 19

For $x > 0$ we have $\sqrt[3]{x}+x^3 > \sqrt[3]{x}$. Therefore

$$\int_0^1 \frac{1}{\sqrt[3]{x}+x^3}dx < \int_0^1 \frac{1}{\sqrt[3]{x}}dx = \lim_{a\to 0}\int_a^1 \frac{1}{\sqrt[3]{x}}dx = \lim_{a\to 0}\frac{x^{2/3}}{2/3}|_a^1 = \lim_{a\to 0}\frac{3}{2}(1-a^{2/3}) = 3/2.$$

4.7, Exercise 21

For $x > 0$ we have $\sqrt[3]{x}+x^3 > x^3$. Therefore

$$\int_1^\infty \frac{1}{\sqrt[3]{x}+x^3}dx < \int_1^\infty \frac{1}{x^3}dx = \lim_{T\to\infty}\int_1^T \frac{1}{x^3}dx = \lim_{T\to\infty}\frac{-1}{2x^2}|_1^T = \lim_{T\to\infty}\frac{1}{2}\left(1-\frac{1}{x^2}\right) = 1/2.$$